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Relationship between Trigonometry Functions with Hyperbolic Function

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ABSTRACT

Many engineering problems can be solved by methods involving complex numbers and complex functions. In the definitions below we will prove the relationship between trigonometric functions and hyperbolic functions, where the hyperbolic function is an extension of the trigonometric function.
Keywords: trigonometric functions; hyperbolic functions

INTRODUCTION

Background

Trigonometric complex functions are defined so that, if the free variable is real value, then the function becomes identical to the known real function.

As ex, which is an ex extension to complex numbers, in the theorems below it will be explained that complex trigonometric functions are also extensions of real trigonometric functions.

eix = cos x + i sin x
e-ix = cos x – i sin x

Through an addition and subtraction, it will be obtained:

cos x = $\frac{1}{2}$ (eix – e-ix)
sin x = $\frac{1}{2i}$ (eix – e-ix)

with x a real number.

Hypothesis

According to Erwin Kreyszig(1), with the definition of complex numbers z = x + iy obtained:

cos z = $\frac{1}{2}$ (eix+ e-ix)
cos z = $\frac{1}{2i}$ (eix – eix)   (1)

defined :

tan z = $\frac{\sin z}{\cos z}$ and cotan z = $\frac{\cos z}{\sin z}$ (2)
next :
sec z = $\frac{1}{\cos z}$ and cosec z = $\frac{1}{\sin z}$ (3)

Because ex is comprehensive, cos z and sin z are also comprehensive, both are analytic except diatitics which make cos z equal zero.

For cotan z and cosec z analytic, except at the point that makes sin z equal zero.

From (ez) '= ez and from equations (1), (2) and (3), derived derivative formulas are as follows:

(cos z)' = - sin z
(sin z)' = cos z
(tan z)' = sec2 z   (4)

From equation (1) that Euler's formulas apply to complex numbers.

eiz = cos z + i sin z   (5)

Real values and imaginary values of cos z and sin z are needed in calculating trigonometric functions. According to Grimaldi, Ralfh P[2]:

cos z = cos x cosh y – i sin x sinh y
sin z = sin x cosh y + i cos x sinh y (6)

and

│cos z│2 = cos2 x + sinh2 y
│sin z│2 = sin2 x + sinh2 y (7)

According to Hungerford [3], cosines and complex hyperbolic sines are defined as follows:

cosh z =$\frac{1}{2}$ (ez + e-z)
sinh z = $\frac{1}{2}$(ez – e-z) (8)

derivatives of the above functions are :

(cosh z)' = sinh z
(sinh z)' = cosh z (9)

The definition of other hyperbolic functions is :

tanh z = $\frac{\sin h\ z}{\cos h\ z}$
cotanh z = $\frac{\cos h\ z}{\sin h\ z}$
sech z = $\frac{1}{\cos h\ z}$
cosech z = $\frac{1}{\sin h\ z}$ (10)

According to Seymour Lipschertz[4], complex trigonometric and hyperbolic functions are interrelated. If in equation (8) we replace z with iz, and by using equation (1), it will be obtained:

cosh iz = cos z
sinh iz = i sin z (11)

Because the zosh is even and sinh z is odd, it is also obtained:

cos iz = cosh z
sin iz = i sinh z (12)

According to Spigel, Murray[5], that the definition of a complex number as a number in the form of a + bi. Symbols for complex numbers are written as z. So z = a + bi with a and b real numbers and i = $\sqrt{-i}$. And a is also called the real part z and is often written with: for example z = x + yi, z1 x1 + y_1^i and z2 = x2 + y_2^i then there are similarities between two complex numbers: z1 = z2 and only if x1 = x2 and y1 = y2.

Hyperbolic function identities

cosh2x –sin h2 x = 1
tanh2x + sech2x = 1
cotanh2x – cosech2x = 1
sinh (x ± y) = sinh x cosh y ± cosh x sinh y
Cosh (x ± y) = cosh x cosh y ± sinh x sinh y
Sinh 2x = 2 sinh x cosh x
Cosh 2x = cosh2x + sinh2
= 1+2 sinh2 x
= 2 cosh2x - 1
Sinh + sinh y = 2 sinh $\frac{1}{2}(x + y)$ cosh$\frac{1}{2}(x - y)$
Sinh x – sinh y = 2 cosh $\frac{1}{2}(x + y)$ sinh $\frac{1}{2}(x - y)$
Cosh x + cosh y = 2 cosh $\frac{1}{2}(x + y)$ cosh $\frac{1}{2}(x - y)$
Coshx – cosh y = 2 sinh$\frac{1}{2}(x + y)$ sinh $\frac{1}{2}(x - y)$

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Identities of the Trigonometry function

sin (x ± y) = sin x cos y ± cos x sin y
cos (x ± y) = cos x cos y ∓ sin x sin y
tan (x ± y) = $\frac{\tan x \pm \tan y}{1 - \tan x \tan y}$
sin2 x + cos2 x = 1
tan2x + 1 = sec2 x
cotan2 x +1 = cosec2 x
sin 2x = 2 sin x cos x
cos 2x = cos2 x – sin2 x
= 1 – 2 sin2 x
= 2 cos2 -1

tan 2x = $\frac{2 \tan x}{1 - \tan^2 x}$
sin 3x = 3 sin x – 4 sin2 x
cos 3x = 4 cos2x – 3 cos x
tan 3x = $\frac{3 \tan x - \tan^2 x}{1 - 3\tan^2 x}$
2 sin x cos y = sin (x + y) + sin (x – y)
2 cos x sin y = sin (x + y) – sin (x -y)
2 cos x cos y = cos (x + y) + cos (x – y)
-2 sin x sin y = cos (x +y) – cos (x - y)
sin x + sin y = 2 sin $\frac{1}{2}$ (x + y) cos $\frac{1}{2}$ (x - y)
sin x – sin y = 2 cos $\frac{1}{2}$ (x + y) sin $\frac{1}{2}$ (x - y)
cos x – cos y = 2 sin $\frac{1}{2}$ (x + y) sin $\frac{1}{2}$ (x - y)
cos x – cos y = 2 sin $\frac{1}{2}$ (x + y) sin $\frac{1}{2}$ (x - y)
sin (-x) = - sin x
cos (-x) = cos x
tan (-x) =– tan x
cotan (-x) = - cotan x

METHODS

The z = x + yi complex number can be seen as a vector in a flat plane with a starting point (0,0) and end point (x, y). fields to describe a complex number are called argand fields. The field of argand is similar to the xy plane, but in the field of argand the x-axis is called the real axis and the y axis is called an imaginary or imaginary axis

Absolute nature of prices

|z| = |-z|
|z-w| =|w-z|
|z|2 = |z2| = zz
|zw|=|z||w|
$|\frac{z}{w}| = \left|\frac{|z|}{|w|}\right|$
|z+w| ≤ |z| + |w|
||z|-|w|| ≤ |z-w|
||z|-|w|| ≤ |z+w|

Complex number operations

Addition

z1± z2 = (x1 ± x2) + (y1 - y2)i

Multiplication

z1 z2 = (x1 x2 - y1 y2) + (x1 y2 - x1 y2) i

Reverse or inverse
The opposite of z is z-1 which has the character z-1 = 1. By doing calculations, you will get it

$$z\text{-}1 = \frac{x}{x^2+y^2} - \frac{y}{x^2+y^2}\, i$$

Division

$$\frac{z_1}{z_2} = \frac{x_1x_2 + y_1y_2}{(x_2)^2 + (y_2)^2} + \frac{x_2y_1 + x_1y_2}{(x_2)^2 + (y_2)^2}\, i$$

The sequel of Z is :

z = x - yi

Table 1. Special elements of complex numbers

No	Special elements of complex numbers	Information
1	Zero Element	The zero element complex number collides 0 ± 0i
2	Unit Elements	Elements of complex number units are shaped1 ±0i
3	Negative element	Negatively shaped element -z = -x - yi

The nature of real numbers which do not exist in the nature of complex numbers is ordered properties. If z and w are two complex numbers, there is no meaning of the symbol z, w

Definition of Argument

Z The z argument written in arg z is defined as the angle formed by the vector z to the positive real axis in an anticlockwise direction.

devinisi means that arg z is the angle θ so that tan $\theta = \frac{y}{x}$ or arg z = θ = arc tan $\left(\frac{y}{x}\right)$ because of the periodic nature of the hand, then the main value of arg z written with arg z is 0 < arg z <2π. So arg z = arg t + 2 kπ

specifically for x = 0 will be obtained arg z = $\frac{\pi}{2}$+ 2kπor$\frac{3\pi}{2}$+ 5 kπ

Distance between two complex numbers
Because of modulus |z| is the distance between o and z then the distance between z with w is |z-w| if z = a+bi and w = c + di, then :

$$|z-w| = |(a+bi) - (c + di)| = \sqrt{(a-c)^2(b-d)^2}$$

Polar shape z
Note the complex number z = y + yi in the argand field. The end point of a complex number z, p: (x, y) can also be seen as a point at the polar coordinates r andθ, so that x = r cos θand y = r sin θ, with r = |z| = $\sqrt{x^2 + y^2}$and θ = arc tan $\left(\frac{x}{y}\right)$, so the complex number z is :

$$Z = 2\left(\cos\left(\frac{4\pi}{3} + 1\right)\sin\left(\frac{4\pi}{3}\right)\right) \text{or } z = 2\cos\left(\frac{4\pi}{3} + 2k\pi\right)$$

Determine Root
Before we can accurately determine complex number equations with complex c numbers, we must know the operation of complex numbers in polar forms
For example :

if z = r cos θ ; z1 = r1 cis $\theta_1$and z2 = r2 cis θ2, then
z1 = z2 r1 = r2and $\theta_1 = \theta_2 + 2k\pi$, k integers
z1 z2 = r1 r2 cis $(\theta_1 + \theta_2)$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\,\text{cis}\,(\theta_1 + \theta_2)$$

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RESULT AND DISCUSSION

Examples and evidences
Examples 1
Prove it : cos z = cos x cosh y – I sin x sin h y
Settlement :
From the equation (1)

$$\text{Cos } z = \frac{1}{2}\left(e^{i(x+y)} + e^{-i(x+y)}\right)$$
$$= \frac{1}{2}e^{-y}(\cos x + i\sin x) + \frac{1}{2}e^{-y}(\cos x - i\sin x)$$
$$= \frac{1}{2}(e^{y} + e^{-y})\cos x - \frac{1}{2}(e^{y} - e^{-y})\sin x$$

This results in (b a), because :

$$\text{Cosh } y = \frac{1}{2}(e^{y} + e^{-y})$$
$$\text{Sinh } y = \frac{1}{2}(e^{y} - e^{-y})$$

Examples 2
Prove it : sin z = sin x cosh y + I cos x sin h y
Settlement :
From the equation (1) :

$$\text{Sin } y = \frac{1}{2}(e^{y} - e^{-y})$$
$$= \frac{1}{2}e^{-y}(\cos x + i\sin x) + \frac{1}{2}e^{y}(\cos x - i\sin x)$$
$$= \frac{1}{2}(e^{y} + e^{-y})\cos x - \frac{1^{i}}{2}(e^{y} - e^{-y})\sin x$$

This produces an equation (b, b), because :

$$\text{Cosh } y = \frac{1}{2}(e^{y} + e^{-y})$$
$$\text{Sinh } y = \frac{1}{2}(e^{y} - e^{-y})$$

Examples 3
Prove it: |cos z|2 = cos2x + sinh2y
Settlement :
From the equation (6a) and cash2y = 1 + sinh2y, then obtained : |cos z|2 = cos2 x (1+sinh2 y) + sin2 x sinh2 y
Because sin2 x + cos2 x = 1 then the equation is produced (7a)

Examples 4
Prove it : |sin z |2 = sin2 x + sinh2 y
Settlement :
From the equation (bb) and cos2 y = 1 + sinh2 y, then obtained :
|sin z|2 = sin2 x (1+cosh2y) = 1 + sinh2 y, then obtained :
|sin z|2 = sin2 x (1+cosh2 y) + cos2 x cosh2 y
Because sin2 x + cos2x = 1 then the equation is produced (7b)

Examples 5
Complete cos z = 5, does not have a real number solution
Settlement :
e2ix – 10eix+ 1 = 0
from equation (1) by multiplying eix, which is the inner quadratic equationeix, then the equation is obtained :
eix = e-y + ix

$$= 5 \pm \sqrt{25-1}$$
$$= 5 \pm \sqrt{24}$$
$$= 5 \pm 4{,}899$$

= 9,899 or 0,101
so e-y = 9,899 or 0,101
eix = 1
y = ± 2,292
x = 2 π (n = 0, 1, 2, 3, ….)

Examples 6
Prove it : cos z = 0
Settlement :
cos x = 0 ; sinh y = 0 according to (7a), so that y = 0
so :
Z = $\pm \frac{1}{2}(2n+1)\pi$ with n = 0,1,2,3,….
So, the value zero cos z is the value of zero cosine functions of real numbers

Examples 7
Prove it : sin z = 0
Settlement :
Sin x = 0 ; sinh y = 0 According to Eq (7b), so that y = 0
so :
Z = 2 n π with n = 0, 1, 2, 3, ….
So, the value of zero sin z is the value of zero sine functions of real numbers
Based on the evidence above, all the trigonometric function formulas also apply to complex numbers
cos (z1$\pm z_2$) = cos $z_1$cos $z_2 \mp \sin z_1 \sin z_2$
sin (z1$\pm z_2$) = cos $z_1$cos $z_2 \mp \sin z_2 \sin z_1$……………………….(13)
dan
cos2 z + sin2 z =1 ……………………………………………….(14)

Complex Trigonometric and Hyperbolic Functions are interrelated
If in equation (8) we replace z with iz, and by using equation (1), it will be obtained :
Cos h iz : cos z
Sin h iz : I sin z ……………………………………………….(15)
Because cos h z is even and sin h z is odd, it is also obtained :
Cos iz : cos h z
Sin iz : i sin h z ……………………………………………….(15)
Examples 8
Determine the real numbers x and y so :
3x + 3iy + 2ix – 2y = 7 + 9 i
Settlement :
3x + 3iy +2ix – 2y = 3x – 2y + (2x+3y)I = 7 + 9i
There are 2 equations namely :
3x – 2y = 7and 2x + 3y = 9
Both equations are eliminated:

3x - 2y = 7	x 2	6x – 4y = 14
2x + 3y = 9	x 3	6x + 9y = 27 -
		- 13 y = -13
		y = 1

so that it is obtained x = 3

CONCLUSION

If in the calculus, the trigonometric function and the hyperbolic function are very different in nature, but in complex systems, both are very closely related to the evidence above, it can be seen that complex trigonometric functions are extensions of real trigonometric functions

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