

**The Application of The Use Integration Technique in Solving An Integral Problem**

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**ABSTRACT**

There are many integration techniques that can be used to solve an integral problem. In this material, several integration techniques will be discussed including substitution, the properties of algebra and trigonometry and techniques commonly used in test books. If we are faced with integral problems that cannot be solved by techniques, what we have learned, we use partial integral techniques. Partial integration technique is obtained by integrating the derivative formula of the product by two functions.

**Keywords:** integration technique; substitution; partial integration technique

**INTRODUCTION**

**Background**

Integration technique is a technique used in integrating functions that are done by changing the form of the integrand into the form contained in the basic integral formula. There are several integration techniques that can be used to solve problems. Integral is substitution, using algebraic operations, with the identity of trigonometry. If the integral problems above cannot be solved by techniques that have been learned, then we can use partial integrals.

**Question**

How to apply the use of integration techniques in solving an integral problem?

**Purpose**

The purpose of this problem is to get a solution about the application of the use of integration techniques in solving an integral problem, to be able to make it easier to understand other materials.

**Literature Review**

According to Hungerford<sup>(1)</sup>, Integration techniques by changing to the basic form of substitution, for example given the integral problem  $\int f(x) dx$ , where the integral of the integrand function is not found in basic integral formula, then in this substance technique is carried out a transformation (substitution) variable  $x$  to another variable (for example variable  $t$ ) such that  $\int f(x) dx = \int g(t) dt$ , where  $\int g(t) dt$  is in the basic integral formula. In addition to transforming variables, a integrator must also be transformed from  $dx$  to  $dt$ ,

To get the relationship  $dx$  and  $dt$  can be obtained by lowering  $x$  with respect to  $t$ .

According to Millington T. Alaric<sup>(2)</sup>, the integration technique using algebraic operations, is meant with algebraic operations in this integration technique is comp and multiplication of  $t$  shape.

- Completing squares is used to solve integrals whose integrations contain the form of quadratic functions  $ax^2 + bx + c$  where  $a$ ,  $b$ , and  $c$  are constants and are not contained basic integral formula. Thus, one technique that can be used is by change the shape of the square to be a perfect square.

$$ax^2 + bx + c = \left(x + \frac{b}{2a}\right)^2 + \left(\frac{b^2 - 4ac}{4a}\right)$$

According to Seymour Lipschutz<sup>(3)</sup>, an integration technique for the Trigonometry Integration Function, where the integrand is a trigonometric function in the form:

- a.  $\sin^m x \cos^n x$
- b.  $\tan^m x \sec^n x$
- c.  $\cot^m x \operatorname{cosec}^n x$
- d.  $\sin mx \cos nx$
- For integrals in the form of  $\sin^m x \cos^n x$  you can use the following substitution:
  1. If m is odd, then substitute  $t = \cos x$
  2. If n is odd, then substitute  $t = \sin x$
  3. If m is even and n even, then Trigonometry identity is used:

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

According to Ayres<sup>(4)</sup> integration techniques for integrating irrational functions, such as integrals containing irrational functions  $\sqrt[n]{f(x)}$  with  $f(x)$  linear functions,  $f(x) = ax + b$ , with a and b constants, then substitution can be used so that a rational function can be used.

According to Spingel<sup>(5)</sup> partial integration techniques, if the integration problem cannot be solved by the techniques that have been learned, we can use partial integration techniques.

This technique is obtained by integrating the two function multiplied derivative formula.

For example  $u = u(x)$  and  $v = v(x)$  functions that are differentiable in  $x$ .

$$\frac{d}{dx} uv = u \frac{dy}{dx} + v \frac{dy}{dx}$$

Partial integral technique:

$$\int u dx = uv - \int v du$$

According to Grimaldi<sup>(6)</sup>, integration techniques by integrating the Partial Fraction function suppose that the integrand is in the form of  $\int \frac{P(x)}{Q(x)}$  where  $P(x)$  and  $Q(x)$  are polynomial functions (terms many) in  $x$  with degrees  $Q(x)$ , to solve the problem, the integrand must be expressed as the sum of partial fractions. In this case we need to describe  $Q(x)$  in linear factors and quadratic factors positive or negative definite.

## METHODS

Integral Formulas:

$$\int a dx = ax + c ; a = \text{konstanta}$$

1.  $\int x^n dx = \frac{1}{n+1} x^{n+1} ; n \neq -1$
2.  $\int e^x dx = e^x + c$
3.  $\int a^x dx = \frac{a^x}{\ln a} + c ; a = \text{positif}$
4.  $\int \frac{1}{x} dx = \ln |x| + c ; x \neq 0$
5.  $\int \sin x dx = -\cos x + c$
6.  $\int \cos x dx = \sin x + c$
7.  $\int \sec^2 x dx = \tan x + c$
8.  $\int \operatorname{cosec}^2 x dx = -\cot x + c$
9.  $\int \sec x \tan x dx = \sec x + c$
10.  $\int \operatorname{cosec} x \cot x dx = -\operatorname{cosec} x + c$

Apart from the formulas above, there are some integral formulas for functions that are not contained in basic integral formula, which can later be sought with integration techniques. There are several integration techniques that can be used to solve problems Integral.

In this material will be discussed, among others:

1. Change to the basic form of substitution

The integral problem  $\int f(x) dx$ , where the integral of the integrand function is not found in the integral formula, then it can be done with the substitution technique variable  $x$  to variable another, for example the variable  $t$ , so that  $\int f(x)dx = \int g(t)dt$  where  $\int g(t)dt$  is found in basic integral formula. Also the integrator differential substitution must be done from  $dx$  to  $dt$ , to get the relationship between  $dx$  and  $dt$  can be obtained by decreasing  $x$  towards  $t$ .

2. Integration technique using algebraic operations

The algebraic operations in question are:

Completing squares is used to solve integrals that are integrands contains the quadratic function form  $ax^2 + bx + c$  with  $a, b$ , and  $c$  a constant and not in the basic integral formula. One technique that can be used is by changing the quadratic shape into a perfect quadratic form.

$$ax^2 + bx + c = \left(x + \frac{b}{2a}\right)^2 + \left(\frac{b^2 - 4ac}{4a}\right)$$

3. Integration technique for Trigonometry function integrals for Trigonometry function integrals Shaped  $\sin^m x \cos^n x, \tan^m x \sec^n x, \cot^m x \csc^n x$  and  $\sin m x \cos n x$ . For integrals in the form of  $\sin^m x \cos^n x$  you can use the following substitution:

- a.  $m$  odd  $\rightarrow$  is used as substitution  $t = \cos x$
- b.  $n$  odd  $\rightarrow$  is used as substitution  $t = \sin x$
- c.  $m$  even

$\rightarrow$  is used with Trigonometry identity, namely:

$n$  even

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

4. Integration technique for integrand Iris integrals that contain functions Irisional  $\sqrt[n]{f(x)}$  with  $f(x) = ax + b$ , where  $a$  and  $b$  are constants, then to solve can substitute be used so that it becomes a function.

5. Trigonometry Substitution Techniques

6. Integran containing one of the irrational forms  $\sqrt{a^2 - x^2}, \sqrt{x^2 - a^2}$  dan  $\sqrt{a^2 + x^2}$  with a arbitrary constant. In this technique trigonometric substitution is performed so the irrational form becomes a rational form.

7. Trigonometric substitution for the irrational form above is as follows:

Table 1. Trigonometric substitution for the irrational form

shape	Substitution
$\sqrt{a^2 - x^2}$	$x = a \sin t$
$\sqrt{x^2 - a^2}$	$x = a \sec t$
$\sqrt{a^2 + x^2}$	$x = a \tan t$

8. Partial Integral Technique

$$\int u dv = uv - \int v du$$

To use partial integral techniques, the following things need to be considered:

- a. The use of this technique is to declare an integrand in 2 parts, a part one as  $u$  and the rest together with  $dx$  as  $dv$ .

- b. When doing step 1, select the dv section that can be integrated.
  - c. It should be considered that  $\int v du$  is no more difficult than  $\int u dv$ .
- Integration on the right hand side is carried to the left hand side, so that:

$$\begin{aligned}
 \int e^x \sin x \, dx &= uv - \int v \, du \\
 \int e^x \sin x \, dx &= e^x (-\cos x) - \int -\cos x e^x \, dx \\
 &= -e^x \cos x + \int e^x \cos x \, dx \\
 \int e^x \cos x \, dx &= e^x \sin x - \int e^x \sin x \, dx \\
 \int e^x \sin x \, dx + \int e^x \sin x \, dx &= -e^x \cos x + e^x \sin x + c \\
 2 \int e^x \sin x \, dx &= -e^x \cos x + e^x \sin x + c \\
 \int e^x \sin x \, dx &= \frac{1}{2} (-e^x \cos x + e^x \sin x) + c
 \end{aligned}$$

Integration technique by making integrals a partial fraction function the ran  $\int \frac{P(x)}{Q(x)} \, dx$  integral with P (x) and Q (x) is the Polynomial function (term many) in x with degrees P (x) smaller than degrees Q (x), to be able to to solve this problem, the integrand must be expressed as a sum of Partial fractions.

If known  $\int \frac{P(x)}{Q(x)} \, dx$  with P (x) and Q (x)

Polynomial function (many terms) then:

a. It must be checked that the degree P (x) is better than the degree Q (x).

b. Describe Q (x) in the positive / negative squared factors.

If Q (x) consists of nonrepeating / different linear factors.

For example Q (x) can be factored like the following form:

$$Q(x) = (a_1 x + b_1) (a_2 x + b_2) \dots \dots \dots (a_n x + b_n)$$

Then  $\frac{P(x)}{Q(x)}$  can be expressed as the following partial fraction:

$$\frac{P(x)}{Q(x)} = \frac{A_1}{(a_1 x + b_1)} + \frac{A_2}{(a_2 x + b_2)} + \dots \dots \dots + \frac{A_n}{(a_n x + b_n)}$$

If Q (x) contains a recurring linear factor.

Then Q (x) can be factored into the following form:

$$Q(x) = (a_1 x + b_1)^n (a_2 x + b_2)$$

Where the linear factor  $(a_2 x + b_2)$  is a non-repeatable linear factor, then it is stated as the following partial fractions:

$$\frac{P(x)}{Q(x)} = \frac{A_1}{(a_1 x + b_1)^n} + \frac{A_2}{(a_2 x + b_2)^{n-1}} + \dots \dots \dots + \frac{A_n}{(a_1 x + b_1)} + \frac{A_n}{(a_2 x + b_2)}$$

## RESULTS AND DISCUSSION

### Examples and Proof

#### Example 1

Prove:  $\int \frac{1}{\sqrt{4-x^2}} dx = \arcsin \frac{x}{2} + c$

#### **Settlement:**

For example:  $y = x / 2$

$$dx = 2 dy$$

$$\begin{aligned} \int \frac{1}{\sqrt{4-x^2}} dx &= \int \frac{1}{\sqrt{1-\frac{x^2}{4}}} \\ &= \frac{1}{2} \int \frac{1}{\sqrt{1+(\frac{x^2}{4})}} \\ &= \frac{1}{2} \int \frac{1}{\sqrt{1-y^2}} \cdot 2 dy \\ &= \int \frac{1}{\sqrt{1-y^2}} dy \\ &= \arcsin y + c \\ &= \arcsin \frac{x}{2} + c \end{aligned}$$

So Proven.

#### Example 2

Prove:  $\int \sin 2x \cos 4x dx = -\frac{1}{12} \cos 6x + \frac{1}{4} \cos 2x + c$

#### **Settlement :**

$$\begin{aligned} \sin 2x \cos 4x &= \frac{1}{2} \left[ \sin (2x+4x) \sin(2x-4x) \right] \\ &= \frac{1}{2} \left[ \sin 6x + \sin (-2x) \right] \\ &= \frac{1}{2} (\sin 6x - \sin 2x) \\ \int \sin 2x \cos 4x dx &= \int \frac{1}{2} (\sin 6x - \sin 2x) dx \\ &= \frac{1}{2} \int \sin 6x dx - \frac{1}{2} \int \sin 2x dx \\ &= -\frac{1}{12} \cos 6x + \frac{1}{4} \cos 2x + c \end{aligned}$$

So Proven.

Example 3

Prove:  $\int \sqrt{4 - x^2} dx = 2 \arcsin \frac{x}{2} + \frac{1}{2} x \sqrt{4 - x^2} + c$

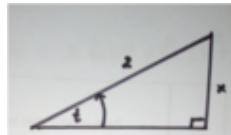
The solution:

$$\int \sqrt{4 - x^2} dx = \int \sqrt{2^2 - x^2} dx$$

**Substitution :**

$$x = 2 \sin t \rightarrow \sin t = \frac{x}{2}$$

$$dx = 2 \cos t dt \rightarrow t = \arcsin \frac{x}{2}$$



$$\sin t = \frac{x}{2}$$

$$\cos t = \frac{\sqrt{2^2 - x^2}}{2}$$

$$\sqrt{2^2 - x^2}$$

$$2 \cos t = \sqrt{2^2 - x^2}$$

$$2^2 \cos^2 t = 2^2 - x^2$$

So :

$$\begin{aligned} \int \sqrt{4 - x^2} dx &= \int \sqrt{2^2 - x^2} dx \\ &= \int \sqrt{2^2 \cos^2 t} \cdot (2 \cos t dt) \\ &= 4 \int \cos t \cos t dt \\ &= 4 \int \cos^2 t dt \\ &= 2 \int \frac{1}{2} (1 + \cos 2t) dt \\ &= 2 \int dt + 2 \int \cos 2t dt \\ &= 2t + 2 \sin 2t + c \\ &= 2 \arcsin \frac{x}{2} + \frac{1}{2} x \sqrt{4 - x^2} + c \end{aligned}$$

So Proven.

Example 4

Prove:  $\int \frac{4x^2+6}{x^3+3} dx = 2 \ln |x| + \ln |x^2+3| + c$

**Settlement :**

$$\int \frac{4x^2+6}{x^3+3x} dx = \int \frac{4x^2+6}{x(x^2+3)} dx$$

Information :

$$\frac{4x^2+6}{x(x^2+3)} = \frac{A}{x} + \frac{Bx+C}{x^2+3}$$

$$\frac{4x^2+6}{x(x^2+3)} = \frac{A(x^2+3) + (Bx+C)x}{x(x^2+3)}$$

$$\frac{4x^2+6}{x(x^2+3)} = \frac{Ax^2 + 3A + Bx^2 + Cx}{x(x^2+3)}$$

$$4x^2 + 6 = A(x^2 + 3) + (Bx + C)x$$

The coefficient  $x^2$  is :  $A + B = 4$

The coefficient  $x$  is :  $C = 0$

The coefficient  $x^0$  is :  $3A = 6$

So:  $A = 2$ ;  $B = 2$ ;  $C = 0$

$$\int \frac{4x^2+6}{x^3+3x} dx = \int \frac{2}{x} dx + \int \frac{2x}{x^2+3} dx$$

So Proven.

**Example 5**

Prove:  $\int \frac{(x^2-2)}{x^3(x+2)^2} dx = -\frac{1}{8} \ln|x| - \frac{1}{2x} + \left| \frac{1}{4x^2} \right| + \frac{1}{4(x-2)} + c$

**Settlement :**

$$\int \frac{(x^2-2)}{x^3(x+2)^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x+2} + \frac{E}{(x+2)^2}$$

So:  $A = -\frac{1}{8}$ ;  $B = \frac{1}{2}$ ;  $C = -\frac{1}{2}$ ;  $D = \frac{1}{8}$ ;  $E = -\frac{1}{4}$

$$\begin{aligned} \int \frac{(x^2-2)}{x^3(x+2)^2} dx &= -\frac{1}{8} \int \frac{1}{x} dx + \frac{1}{2} \int \frac{1}{x^2} dx - \frac{1}{2} \int \frac{1}{x^3} dx \\ &\quad + \frac{1}{8} \int \frac{1}{(x+2)} dx - \frac{1}{4} \int \frac{1}{(x+2)^2} dx \\ &= -\frac{1}{8} \ln|x| - \frac{1}{2x} + \left| \frac{1}{4x^2} \right| + \frac{1}{4(x+2)} + c \end{aligned}$$

So Proven.

**Example 6**

Prove:  $\int x^2 \ln x dx = \frac{1}{4} x^4 \ln x - \frac{1}{16} x^4 + C$

**Settlement :**

For example:

$u = \ln x \quad dv = x^2 dx$

$du = \frac{1}{x} dx \quad v = \frac{1}{4} x^4$

$$\begin{aligned} \int x^2 \ln x dx &= uv - \int v du \\ &= (\ln x) \cdot \left(\frac{1}{4} x^4\right) - \int \frac{1}{4} x^4 \cdot \frac{1}{x} dx \\ &= \frac{1}{4} x^4 \ln x - \frac{1}{4} \int x^3 dx \\ &= \frac{1}{4} x^4 \ln x - \frac{1}{16} x^4 + C \end{aligned}$$

So Proven.

Example 7.

Prove it :  $\int \sin^5 x \cos^3 x \, dx = \frac{1}{6} \sin^6 x - \frac{1}{8} \sin^8 x + C$

**Settlement :** 
$$\begin{aligned} \int \sin^5 x \cos^3 x \, dx &= \int \sin^5 x \cdot \cos^2 x \cdot \cos x \, dx \\ &= \int \sin^5 x (1 - \sin^2 x) \, d(\sin x) \\ &= \int \sin^5 x \, d(\sin x) - \int \sin^7 x \, d(\sin x) \\ &= \frac{1}{6} \sin^6 x - \frac{1}{8} \sin^8 x + C \end{aligned}$$

So Proven.

**CONCLUSION**

There are two important things that need to be considered in solving integral problems, vizan integral problem sometimes requires more than one deep and integration technique settlement there is never a particular sequence which technique is used first first So the two statements are very important for us to master all techniques thoroughly and how to integrate all of these techniques.

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