AIJMu
Aloha International Journal of Multidisciplinary Advancement (AIJMU)
ISSN 2622-3252
RESEARCH
Volume 4 Number 2, February 2022
DOI:
http://journal.aloha.academy/index.php/aijmu
http://dx.doi.org/10.33846/aijmu40201

Determining the Roots Equation of Characteristics

Sri Rejeki Dwi Putranti[1] (corresponding author), Suroso Ari Purnomo[2]
[1]Faculty of Engineering, Universitas Yos Soedarso Surabaya, Indonesia; riccayustisia@gmail.com
[1]Faculty of Engineering, Universitas Yos Soedarso Surabaya, Indonesia

Submitted: January 2, 2022 -Revised: February 10, 2022 -Accepted: February 15, 2022 -Published: February 28, 2022

ABSTRACT

Characteristic equation an matrix roots called characteristic roots. Here is discussed how to determine the roots of the characteristic of several kind if matrices. Such us Matrix Hermitian, Orthogonal Real Matrix and Matrix Similar.The roots of a Hermition matrix and the characteristic roots of a real symmetric matrix must be real. While the roots of an Orthogonal Riel matrix have 1 as mode. For a Similar matrix, the characteristic roots are the same.
Keywords: matrix; root equation; characteristic

INTRODUCTION

Suppose we define a square matrix A = (a_{if}) $type\ n\ x\ n$ with elements from Field F. If we can determine the vector x such that from Ax = x we get:

$$\begin{aligned} Ax = \pi x &= Ax - \pi\, l\, x \\ &= (A - \pi\, l)\, x \\ &= 0 \end{aligned}$$

$$(A - \pi\, i)\, x = \begin{bmatrix} a_{11}-\pi & a_{12} & \ldots & a_{1\pi\, n} \\ a_{21} & a_{22}-\pi \ldots & & a_{2\pi\, n} \\ a_{\pi 1} & a_{\pi 2} & \ldots . & a_{\pi n} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

or:

$$(a_{11} - \pi)\, x_1 + a_{12}\, \gamma_{12} + \cdots + a_{1n}\gamma_n = 0$$
$$a_{21}\gamma_1 + (a_{22} - \pi)\, x_2 + \cdots + a_{2n}\gamma_n = 0$$
.
.
.
$$a_{n1}\gamma_1 + (a_{n2}\gamma_2 + \ldots + a_{nn} - \pi)\, x_n = 0 \quad \ldots\ldots (1)$$

The system of equations (1) has a non-trivial solution if (A-)=0

CHARACTERISTIC ROOTS

Definition of I

Equation $[A - \pi] = f(\pi) = 0$ with as a number or is called equation characteristic of the matrix A. The roots are called characteristic roots.
If $\pi_{1,}, \pi_{2,}, \ldots, \pi_n$ are the roots of the equation [A-πl]=f (π)=0, then we get $[A - \pi_1 l] = 0$, $[A - \pi_2\, l] = 0$. etc. So that the equation $[A - \pi_1 l] = 0$, $[A - \pi_2\, l] = 0$ has a non-trivial solution. Vector characteristics.

Find: Characteristic vectors and matrix A = $\begin{bmatrix} 4 & -6 & 2 \\ -6 & 3 & -4 \\ 2 & -5 & -1 \end{bmatrix}$

Solution:

$$[A - \pi l] = \begin{bmatrix} 4-\pi & -6 & 2 \\ -6 & 3\pi & -4 \\ 2 & -4 & -1-\pi \end{bmatrix} = 0$$

$$= \begin{bmatrix} 4-\pi & -6 & 2 \\ -6 & 3\pi & -4 \\ 2 & -4 & -1-\pi \end{bmatrix} \begin{matrix} 4-\pi & -6 \\ -6 & 3\pi \\ 2 & -4 \end{matrix} = 0$$

$(4-\pi)(3-\pi)(-1-\pi)+(-6)(-4)(2)+(2)(-6)(-4)-(2)(3-\pi)(2)$ -
$(4-\pi)(-4)(-4)-6(-6)(-1-\pi)=0$
$(12-4\pi-3\pi+\pi^2)(-1-\pi)+(24)(2)+(-12)(-4)-(6-2\pi)(2)-$
$(-16+4\pi)(-4)-(36)(-1-\pi)=0$
$(12-7\pi+\pi^2)(-1-\pi)+48+48-(12-4\pi)-(64-16\pi)-(-36-36\pi)=0$
$(-12-12\pi+7\pi+7\pi^2-\pi^2-\pi^3)+96-12+4\pi-64+16\pi+36+36\pi=0$
$(-12-5\pi+6\pi^2-\pi^3)+84-64+20\pi+36+36\pi=0$
$-12-5\pi+6\pi^2-\pi^3+20+20\pi+36+36\pi=0$
$-\pi^3+6\pi^2+51\pi+44=0$
$\pi^3-6\pi^2-51\pi-44=0$ .... (1)
$\pi^3-6\pi^2-51\pi-44=0$
-44 is divisible by ± 1, ± 2, ± 4, ± 11, ± 22, ± 44
If $\pi=-1 \rightarrow \pi^3-6\pi^2-51\pi-44=(-1)^3-6(-1)^2-51(-1)-44$
$= -1-6+51-44$
$= -51+51$
$= 0$

So =-1 → is the root of equation (1)
So (π+1) → is a factor of equation (1)
To find the roots of other equations, we use for less as below this:

$$\begin{array}{r} \pi^2-7\pi-44 \\ (\pi+1)\overline{\smash{)}\pi^3-6\pi^2-51\pi-44} \\ \underline{\pi^3+\pi^2} \quad - \\ -7\pi^2-51\pi-44 \\ \underline{-7\pi^2-7\pi} \quad - \\ -44\pi-44 \\ \underline{-44\pi-44} \quad - \\ 0 \end{array}$$

So: $\pi^3-6\pi^2-51\pi-44=0$
$(\pi+1)(\pi^2-7\pi-44)=0$
Then:
$\pi^2-7\pi-44=0$ .... (2)
-44 is divisible by ± 1, ± 2, ± 4, ± 11, ± 22, ± 44
If $\pi=-4 \rightarrow \pi^2-7\pi-44=(-4)^2-7(-4)-44$
$= 16+28-44$
$= 44-44$
$= 0$

So $\pi=-4$ → is the root of equation (2)
So $(\pi+4)$ → is a factor of equation (2)
To find the roots of other equations, we use parentheses, as shown below :

$$\begin{array}{r} \pi-11 \\ (\pi+4)\overline{\smash{)}\pi^2-7\pi-44} \\ \underline{\pi^2+4\pi} \quad - \\ -11\pi-44 \\ \underline{-11\pi-44} \quad - \\ 0 \end{array}$$

Then: $\pi^2-7\pi-44=0$
$(\pi+4)(\pi-`11)=0$
So: $\pi^3-6\pi^2-51\pi-44=0$
$(\pi+1)(\pi+4)(\pi-11)=0$
So the characteristic roots of matrix A are:
$\pi_1=11$ $\quad n_2=-1$ $\quad \pi_3=-4$
Now we will find the characteristic vector according to $\pi_1=11$
Then $[A-11\,l]\,x=0$

Aloha International Journal of Multidisciplinary Advancement (AIJMU)
ISSN 2622-3252
RESEARCH
Volume 4 Number 2, February 2022
DOI:
http://journal.aloha.academy/index.php/aijmu http://dx.doi.org/10.33846/aijmu40201

$$\begin{bmatrix} 4-11 & -6 & 2 \\ -6 & 3-11 & -4 \\ 2 & -4 & -1-11 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Equivalent to the equation:
$-7x_1 - 6x_2 + 2x_3 = 0$ .... (1)
$-6x_1 - 8x_2 - 4x_3 = 0$ .... (2)
$2x_1 - 4x_2 - 12x_3 = 0$ .... (3)

By elimination, we get:
$x_1 = 2$ $x_2 = -2$ $x_3 = 1$

Then the column vector x = (2,-2,1) is obtained as the characteristic vector of the A . matrix which corresponds to $\pi_1 = 11$..
In the same way for _2= -1, we get the column vector x = (2,1,-2).
Likewise, for _3= -1, the vector x = (1,2,2) is obtained.

ROOTS OF CHARACTERISTICS OF THE HERMITIAN MATRIX

Theorem 2

The roots of a Hermitian matrix and the characteristic roots of a matrix real symmetry must be real.
Proof:
If π_1 is the characteristic root of matrix A (hermitian or special symmetric matrix riel) then $|A - \pi_1 l| = 0$

so that $|A - \pi_1 l| x = 0$ has non-trivial

so $A_x = \pi_1 x$ or atau $\bar{x} * Ax = \bar{x} * \pi_1 x * x *$ .... (2)
From $A_x = \pi_1 x$ we get $\overline{Ax} = \overline{\pi_1 x}$ namely:
$\overline{Ax} = \pi_1 x$ after transpose then $\bar{x} * \bar{A} * = \overline{\pi_1} \bar{x} *$
Since A is hermitian or real symmetric, then A * = A so that $\bar{x} + A = \overline{\pi_1} \bar{x} *$
From equation (2), then we get $\overline{\pi_1} \bar{x} *$ or atau $(\overline{\pi_1} - \pi_1) \bar{x} * x = 0$
Because $\bar{x} * x \neq 0$ (x non $-$ trivial) then $\overline{\pi_1} - \pi = 0$ so $\overline{\pi_1}$ and prove π_1 real.

ROOT CHARACTERISTICS OF THE ORTHOGONAL RIEL MATRIX

Theorem 3

The characteristic roots of an orthogonal real matrix have 1 as the mode
Proof:
An orthogonal matrix (by definition always real) may have roots complex characteristics. However, the modulus is equal to 1, for example is the characteristic root of the orthogonal matrix A and x the characteristic vector that according to the root,
Then Ax = πx and $\overline{Ax} = \overline{\pi x}$ or $\overline{Ax} = \overline{\pi x}$
Because Ariel then maka A $\bar{x} = \overline{\pi x}$
So $\overline{\bar{x} * A *} = \bar{x} * \bar{\pi}$
Remember Ax = x then $\bar{x} * A * Ax = \bar{\pi} \pi \bar{x} * x$
Since A * A = l then $\bar{x} * x = \bar{\pi} \pi \bar{x}$ or $\bar{x} * x (l - \bar{\pi} \pi) = 0$
Since $\bar{x} * x \neq 0$ then $\bar{\pi} \pi = |x| = 1$

SIMILAR MATRIC CHARACTERISTICS ROOTS

Theorem 4

The characteristic roots of similar matrices are the same.
Proof:
Suppose matrix B is similar to matrix A then B = $C^1 AC$ or $B = C_1 A C_1^{-1}$
where $C_1 = C^{-1}$

Aloha International Journal of Multidisciplinary Advancement (AIJMU)
ISSN 2622-3252
RESEARCH
Volume 4 Number 2, February 2022
DOI:
http://journal.aloha.academy/index.php/aijmu http://dx.doi.org/10.33846/aijmu40201

so:

$$B - \pi = | C^1 AC - \pi l|$$
$$= | C^{-1}(A - \pi l) C |$$
$$= | C^{-1}| \; |(A - \pi l)| \; | C |$$
$$= | (A - \pi l)$$

It can be seen that the characteristic determinants of the 2 similar matrices are the same.
So the characteristic roots are also the same.

Theorem 5

If the matrix A is similar to the diagonal matrix D the elements of D roots Characteristics of the matrix A.

Proof :

$$D - \pi = \begin{vmatrix} d_1 - \pi & 0 & \dots & 0 \\ 0 & d_2 - \pi & \dots & 0 \\ \vdots & \vdots & & \vdots \\ d_1 - \pi & d_x - \pi & & d_\pi - \pi \end{vmatrix}$$

So the characteristic roots of D are the diagonal elements.
Since the matrix A is similar to D, then the characteristic roots of the matrix A are characteristic roots of D.
So it's proven.

CONCLUSION

Characteristic roots of the hermitian matrix, especially the characteristic roots of the matrix real symmetrical must also be real. The modulus of the characteristic roots of orthogonal real matrices must be equal to one.

REFERENCES

1. Ayres JR. Calculus 2/ed Singapore, Schaum's Outline Series. Mc Graw – Hill International Book Company; 1981.
2. Gilmer R. Zero Devisions in Commulative Rings. Washington: The American Mathematical Monthly; 1986.
3. Hungford TW. Algebra. New York: Sfringer – Verdag; 1984.
4. Lang S. Linear Algebra. Addison – Wesley Publi, Co.; 1981.