Aloha International Journal of Multidisciplinary Advancement (AIJMU)
ISSN 2622-3252
Volume 5 Number 12, December 2023
RESEARCH
http://journal.aloha.academy/index.php/aijmu DOI: http://dx.doi.org/10.33846/aijmu51201

# Application of Differential Equations Separate Variables in Physics

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Submitted: November 3, 2023 -Revised: December 3, 2023 -Accepted: December 4, 2023 -Published: December 4, 2023

## ABSTRACT

Many important problems in engineering, physics and the social sciences, when formed in mathematical form, require the determination of a function that satisfies one or more derivative of an unknown function. Such an equation is called a Differential Equation. One of the differential equations we know is Newton's Law. One obvious clarification is to look at whether an unknown function depends on one or more independent variables, when only one is called a Partial Differential Equation. In this chapter will be discussed Differential Equations with Separate Variables (can be separated) to be able to solve in physics problems.
**Keywords**: equation; variable; differential

## INTRODUCTION

### Background

If there is more than one unknown function, more than one differential equation is required.(1-5) The solution of a differential equation is a relationship between variables without derivatives and those that satisfy the differential equation. General solution of differential equation is the solution of differential equations that contain any constant whose number is equal to the derivative of the differential equation.(6-9) A special solution of a particle of a differential equation is the solution of a differential equation obtained from the general solution of a differential equation if both constants are arbitrarily given a certain price.

### Literature Review

One of the differential equations we know is Newton's Law.(10-15)

$$\frac{d^2u(t)}{dt^2} = F\,[\,L\,u\,(t)\,]\,\frac{du\,(t)}{dt}$$

For the position of a particle u (t) subjected to force F which is a function of time t, position u (t) and velocity of $\frac{(t)}{dt}$. To determine the motion of a particle subjected to the force F, a function u that satisfies the equation is required.

### Ordinary and Partial Differential Equations

One obvious classification is to look at whether an unknown function depends on one or more independent variables. When only one is called a Partial Differential Equation.

1) $L\,\frac{d^2Q(t)}{dt^2} + B.\frac{du(t)}{dt} + \frac{1}{c}\,Q(t) = E(t)$

Q (t) = electric charge

C = capacity

B = prisoner

L = inductor

E (t) = voltage

**2)** $\frac{du(t)}{dt} + k\,R(t) = 0$

**R (t) =** time function, decrease in number radioactive materials, such as **Radium**

**Systems of Differential Equations**

When an unknown function is more than one differential equation.

$$\frac{dH}{dt} = a\,H - \propto HP$$
$$\frac{dP}{dt} = -eP + \gamma\,HP$$

Where H(t) and P(t) are populations of prey species and predator species A, ∝, C, γ are constants based on observation.

**Definition of Differential Equations**

The definition of a differential equation is an equation that contains the derivative of one or more unknown functions. A regular differential equation is a differential equation that contains only one independent variable.

**Levels and Degrees of a Differential Equation**

The level of a differential equation is the highest derivative that looks at the differential equation. And what is meant by the degree of a differential equation is the power of the derivative highest in the differential equation.

$y' + x^{-1}y = 5x$ ........................................................................ (1)
$y'' + 3y' + 4y = 0$ ................................................................ (2)
$(y')^3 + yy' = x$ ........................................................................ (3)

Equation (1) → express differential equations of degree one, degree one
Equation (2) → express differential equations of degree two, degree one
Equation (3) → express differential equations of degree one, degree three of the function y(x) with respect to x.

**Differential Equations with Separate Variables and Breakable Variables**

**(**1) Differential equations with separate variables
In general:

$$f(x)dx + g(y)dy = 0$$

The General Solution of the Differential Equation is:

$$\int f(x)dx + \int g(y)dy = c$$

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(2) Differential equations with variables that can be separated
In general:

$$f(x).v(y)dx + g(x).u(y)dy = 0$$

Divided by the function g(x) .v(y) obtained a differential equation with
Separate variables are:

$$\frac{f(x)}{g(x)}\,dx + \frac{u(y)}{v(y)}\,dy = 0$$

The general solution of the differential equation is:

$$\int \frac{f(x)}{g(x)}\,dx + \int \frac{u(y)}{v(y)}\,dy = 0$$

Example (1):
Solve the differential equation below:

$$\frac{dy}{dx} = \frac{x^2}{1+3y^2}$$

Settlement:

$$\frac{dy}{dx} = \frac{x^2}{1+3y^2}$$

$$\frac{dy}{dx} - \frac{x^2}{1+3y^2} = 0$$

$$(1 + 3y^2)dy - x^2 dx = 0$$

$$\int(1 + 3y^2)dy - \int x^2\,dx = c$$

$$\left(y + 3.\,\frac{1}{3}\,y^3\right) - \frac{1}{3}\,x^3 = c$$

So the general solution of differential equations is:

$$y + y^3 - \frac{1}{3}\,x^3 = c$$

Example (2):
Solve the differential equation below:

$$e^{x-y}y' - \sin x = 0$$

Settlement:

$$e^{x-y}y' - \sin x = 0$$

$$e^{x-y}\,\frac{dy}{dx} - sin\ x = 0$$

$$e^{-y}dy - e^{x}\sin x\,dx = 0$$

$$\int e^{-y}dy - \int e^{x}\sin x\,dx = 0$$

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$$-e^{-y} - \frac{1}{2}e^{x}\cos x = c$$

So the general solution of differential equations is:

$$e^{-y} - \frac{1}{2}e^{x}\cos x = c$$

## METHODS

### Solving A Differential Equation

The solution of a differential equation is a relationship between variables- variables without derivatives and which satisfy the differential equation. General solution of differential equations containing arbitrary constants which is the same as the derivative of the differential equation.

### Linear and Non Linear Differential Equations

Differential Equations
1). F $\left(x, y, y', \ldots, y^{(n)} = 0\right)$

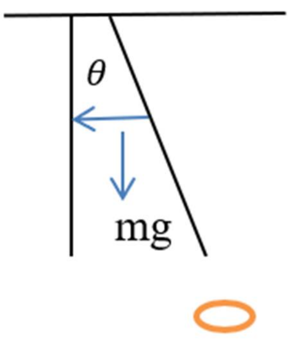
It is called linear when F is a linear function of the variable
$y, y', \ldots, y^{n}$
So the form of differential equations is:

2). $a_0(x)\, y^{(\ldots)} + a_1(x)\, y^{(n-1)} + \ldots + a_n(x) y = g(x)$

So that equations that do not have the form as above are called non-linear
Simple physics problems that form nonlinear differential equations
is Oscillations.

3). $\frac{d^2\theta}{dt^2} + \frac{\theta}{t}\sin\theta = 0$

Theories and techniques for solving linear differential equations
Developed Rapid, on the contrary, for non-linear ones is very limited.
Yet so many Also, the problem of nonlinear differential equations can
be reduced to linear differential equations or their approximations, e.g.
for pendulum problems.




The angle θ is very small, so sinθ = 0,
So that equation (3) becomes:

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4). $\frac{d^2\theta}{dt^2} + \frac{\theta}{t}\,\theta = 0$

Conversely, physical phenomena on ecological problems, for example, are impossible to be bounded by linear differential equations.

**Homogeneous Differential Equation**

The differential equations of level one and degree one are called homogeneous. If the differential equation can be expressed in the form of:

$$\frac{dy}{dx} = \int (yx^{-1}) \qquad (1)$$

Medium $\int(\,x, y\,)$is called homogeneous of degree n

If: $\int(\sigma x, \sigma y) = \sigma^n \int(x, y)$

To solve the differential equation (1) with substitution y = vx,
reduces the differential equation (1) to a separate differential equation.

Example (1):

Solve the differential equation below:

$(x^2 + y^2)dx + 2xy\,dy = 0$

Settlement:

$(x^2 + y^2)dx + 2xy\,dy = 0$

$\frac{dy}{dx} = \frac{x^2+y^2}{2xy} = -\frac{(1+(yx^{-1})^2)}{2yx^{-1}}$

Substitution: y = vx and dy = v dx + x dv
Then the differential equation becomes:

$(1 + 3v^2)x^2\,dx + 2x^3 v\,dv = 0$

$\frac{2v\,dv}{1+3v^2} + \int \frac{dx}{x} = c$

$\frac{1}{3}\int \frac{d\,(1+3v^2)}{1+3v^2} + \int \frac{dx}{x} = c$

$(1 + 3v^2)\,.\,x^3 = c$

$(1 + 3y^2x^2\,)x^3 = c$

So the general solution to the differential equation is:

$$\left(1 + 3\,\frac{y^2}{x^2}\right)x^3 = c$$

Example (2):

Solve the differential equation below:

$\left(e^{\frac{y}{x}} + \frac{y}{x}\right)dx - dy = 0$

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Settlement:

$$\left( e^{\frac{y}{x}} + \frac{y}{x} \right) dx - dy = 0$$

$$( e^{y} + v )dx - ( v\, dx + x\, dv )$$

$$e^{y} dx - x\, dv = 0$$

$$\frac{dy}{x} - \frac{dv}{y} = 0$$

$$\int \frac{dy}{x} - \int e^{-y} dv = c$$

So the general solution to the differential equation is:

$$\ell n\, x + e^{-\frac{y}{x}} = c$$

**Exact Differential Equation**

A differential equation of the form M (x,y) dx + N (x,y) dy = 0 is called an exact differential equation, if there is a function F( x,y ) whose total differential is equal to M (x,y) dx+N (x,y) dy i.e. :

$$dF = \frac{\partial F}{\partial x}\, dx + \frac{\partial F}{\partial y}\, dy$$

$$= M(x, y)dx + N(x, y)dy$$

**Theorem**

The necessary and sufficient conditions for the equation

M(x,y)dx + N(x,y)dy = 0 are

The exact equation is:

$$\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}$$

The general solution of the exact differential equation is of the form :

F(x,y) = c ....................................................................................................................... (1)

Where:

$$\frac{\partial F(x,y)}{\partial x} = M(x, y) \text{ and}$$

$$\frac{\partial F(x,y)}{\partial y} = N(x, y)$$

From the two relationships above, we can find F(x,y) as follows:

From $\frac{\partial F}{\partial x} = M(x, y)$ then:

$$F(x, y) = \int^{x} (x, y)dx + R(y)$$

or

$\frac{\partial F}{\partial y} = N(x, y)$ then:

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$$F(x,y) = \int^{y} N(x,y) + Q(x)$$

Where $\int^{x} M$ states that in the integration y is considered a constant and in this case:
R(y) is the constant of integration
$\int^{y} N(x,y)$ states that in integration x is considered constant and in this case Q(x) is the constant of integration.

Thus: $F(x,y) = \int^{x} Mdx + R(y)$ or

$F(x,y) = \int^{y} N(x,y) + Q(x)$

R(x) or Q(x) is determined as follows:

From equation (1) is obtained:

$$\frac{\partial F}{\partial y} = \frac{d}{dy}\left(\int^{x} Mdx\right) + \frac{dR(y)}{dy} = N(x,y) \quad \text{.......... (2)}$$

Then equation (2) can be found R(y) or Q(x), then
Substitute into equation (2) and we get:

The general solution of the differential equation is:

$$F(x,y) = c$$

Example (3):
Solve the differential equation below:

$(3x^2 + 4xy^2)dx + (2y - 3y^2 + 4x^2y)dy = 0$

Settlement:

$M(x,y) = 3x^2 + 4xy^2$ dan $N(x,y) = 2y - 3y^2 + 4x^2y$

$\frac{\partial M}{\partial y} = 8xy$

$\frac{\partial N}{\partial x} = 8xy$

So, $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = 8xy$

So differential equations are exact

General solution of exact differential equations of the form: F(x,y) = c

$$\begin{aligned} F(x,y) &= \int^{x} M\,dx + R(y) \\ &= \int^{x}(3x^2 + 4xy^2)dx + R(y) \\ &= x^3 + 2x^2y^2 + R(y) \end{aligned}$$

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$$\frac{\partial F}{\partial y} = 4x^2y + \frac{dR}{dy} = 2y - 3y^2 + 4x^2y$$
$$\frac{dR}{dy} = 2y - 3y^2 + 4x^2y$$
$$R(y) = y^2 - y^3$$

So the general solution to the differential equation is :

$$F(x,y) = x^3 + y^2 - y^3 + 2x^2y^2 = c$$

Example (4):

Solve the differential equation below:

$$\left[\cos x \, Ln\,(y-4) + \frac{1}{x}\right] dx + \frac{\sin x}{y-4}\, dy = 0$$

Settlement:

$$M(x,y) = \cos x \;\; Ln(y-4) + \frac{1}{4}$$
$$N(x,y) = \frac{\sin x}{y-4}$$
$$\left.\begin{aligned}\frac{\partial M}{\partial y} &= \frac{\cos x}{y-4}\\ \frac{\partial N}{\partial x} &= \frac{\cos x}{y-4}\end{aligned}\right\} \text{ So } \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = \frac{\cos x}{y-4}$$

So it is an exact differential equation

The general solution to the differential equation is:

$$F(x,y) = c$$
$$F(x,y) = \int^{y} N\, dy + Q(x)$$
$$= \int^{y} \frac{\sin x}{y-4}\, dy + Q(x)$$
$$= \sin x \, Ln(y-4) + Q(x)$$
$$\frac{dy}{dx} = \cos x \;\; Ln(y-4) + \frac{dQ}{dx}$$
$$= \cos x \, Ln(y-4) + \frac{1}{x}$$
$$Q(x) = Ln\, x$$

So the general solution to the differential equation is:

$$F(x,y) = \sin x \, Ln(y-4) + Ln\, x = c$$

## RESULTS AND DISCUSSION

In this chapter, we will discuss the application of differential equations with separate variables in physics.
Solve the problem below:
An object of mass m = 2 kg is thrown down from a high area with velocity with speed $V_0$ =105 cm/sec.
In addition to its weight there is air friction acting on this object, the magnitude of which is (in dynes) twice the velocity of the object at each instant. What is the velocity of the object after $t = 10^3$ seconds. Determined g = 900 cm/$sec^2$.

Solution:

- ❖ Newton's second law regarding the motion of an object with mass m and velocity v is determined by the differential equation:

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$$\frac{d}{dx}(mv) = kF \quad \ldots\ldots (1)$$

Let v(t) be the velocity of the object at time t.
The sum of the forces acting on the falling object:
F = (weight) - (air friction)
= (mg - 2V) dine
According to equation (1), it is obtained:

$$\frac{d}{dt}(m\,v) = mg - 2v$$

$$m\frac{dv}{dt} = mg - 2v$$

$$\frac{m}{mg-2v}\,dv = dt$$

$$v(t) = -\frac{1}{2}e^{-2t/m} + \frac{mv}{2} \text{ dan } \; v(0) = 10^3$$

So:

$$V(10^3) = \left(9 - \frac{8}{e}\right)10^3 \text{ cm/sec}$$

$$= \left(9 - \frac{8}{e}\right) \text{ km/sec}$$

$$= 6{,}05 \text{ km/sec}$$

- Solve the problem below:
  If at the moment the air temperature is $290^0 k$
  An object cools from $370^0 k$ to $330^0 k$ in 10 minutes.
  What is the temperature after 40 minutes?
  Solution:

$$\frac{dT}{T-290^0} = -k\,dt$$

$$\int_{370}^{330}\frac{dT}{T-290^0} = \int_0^{10} -k\,dt$$

$$ln\frac{330-290}{370-290} = -10\,k$$

$$\boxed{10k = Ln\,2}$$

Then : $\text{Ln}\int_{370}^{r}\frac{dT}{T-290^0} = -40k$
$= 4\,Ln\,2$
$= Ln\,\frac{1}{16}$

$$T - 290 = \frac{80}{16} = 5 \;\rightarrow T = 295^0$$

So after 40 minutes the temperature of the object is $295^0$

- Solve the problem below:
  After the tank contains 450 liters of brine made by dissolving 30 kg salt in water.
  Brine containing 1/9 kg of salt per liter is flowed at a rate of 9/minute and stirred

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So it flattens and then flows out with an average of 13.5 liters / minute.
Determine the amount of salt in the tank after 1 hour ?
Completion:
The amount of water in the tank at time t is:

V = 450 - (13.5-9)t

= 450 - 4.5t liters

When x is a lot of salt in the tank at time t is:

$$\frac{dx}{dt} = 1 - \frac{13{,}5\,x}{450-4{,}5\,t}$$

$$= 1 - \frac{3x}{100-t}$$

$$\int \frac{3}{100-t}\,dt = -3\ln(100-t)$$

$$= \ln(100-t)^3$$

❖ Integral Factors are: $\frac{1}{(100-t)^3}$

The general solution of differential equations is:

$$\frac{x}{(100-t)^3} = \int \frac{dt}{(100-t)^3}$$

$$= \frac{1}{2(100-t)^2} + c$$

$$x = \frac{1}{2}(100-t) + c(100-t)^3$$

For t = c → x = 30 obtained:

$$30 = \frac{1}{2}(100-t) + c(100-t)^3$$

$$c = 2\,x\,10^{-5}$$

❖ $x = \frac{1}{2}(100-t) - 2(100-t)^3\,x\,10^5$

❖

For 1 hour = t = 60 minutes

$$x = 20 - 2\;x\;40^3\;x\;10^{-5}$$

$$= 20 - 20(\,0{,}4\,)^3$$

$$= 20 - 1{,}28$$

$$= 18{,}72$$

So after 1 hour the amount of salt in the water = 18.72 kg

## CONCLUSION

The application of applied problems in the field of physics, in the form of mathematics, requires determination of a function that satisfies one or more problems derivatives of unknown functions. One obvious classification is to see if the functions are unknownon one or more independent variables. When only one is called an equation partial differential. When more than one unknown function is required, more than one is required differential equation. So a differential equation with variables that can be separated is an equation that contains derivatives of several functions that are not known.

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